Philosophy of Mathematics

Is Logic A Priori?

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1. Introduction

The received view about logic is that it is a priori, in a double-headed way: (i) the legitimacy of logical rules does not depend upon empirical evidence, and (ii) these rules are empirically indefeasible; that is, there is no possible observational evidence that counts against the legitimacy of these rules. Apriorism about logic is typically tied to a dogmatic attitude: logic is empirically indefeasible because we need logic to draw consequences from empirical observations, and this cannot be done if logic itself is revisable.

In two fascinating papers, Hartry Field has given a new twist to this position (see Field 1996 and 1998; see also Field 2001, Chapter 13). He has argued that the logical apriorist need not be dogmatic. On his view, logical apriorism is consistent with logical fallibilism (even on empirical grounds): “It makes sense to regard logic as a priori but at the same time to think it conceivable that further conceptual developments could show it not to be, by showing that logic is empirically defeasible . . . after all” (Field 1998, p. 4).

Although this is an interesting move, fallibilism and apriorism do not appear to be reconcilable. If we allow the possibility that logic is empirically defeasible, as Field does with his logical fallibilism, we are allowing that logic might not be a priori—contrary to Field’s apriorism. Field tries to establish the coherence of this position by claiming that the possibility of empirical defeasibility he considers is only epistemic. But since in his view epistemic possibility is not genuine possibility (Field 1998, p. 5), it is not at all clear in what sense his fallibilist apriorism is genuine: for the possibility of empirical defeasibility, being only epistemic, is not genuine in Field’s own light.

In what follows, I shall argue that even if we grant that fallibilist apriorism is coherent, Field fails to establish that this proposal is better than the most interesting alternative: fallibilist empiricism. According to this empiricist view,
although the legitimacy of logical rules does not depend upon empirical evidence, logic is empirically defeasible.

In Field’s view, any proposal that supports the non-apriority of logic is inadequate, since it has to assume that some logical rules are a priori in order to evaluate other logical rules and the adequacy of relevant methodological standards. I will resist this argument by developing the empiricist alternative further. The main idea is to articulate a logical pluralist account (according to which there is more than one logic adequate to a given domain), and to motivate the claim that logic is not topic-neutral, but is context-dependent. Depending on the domain we are considering (for instance, quantum mechanics), there may be good reason to revise certain logical rules on empirical grounds (see Birkhoff and von Neumann 1936, Putnam 1968, Haack 1996, and da Costa 1997). The change in the underlying logic may help to solve certain problems (for example, interpretation issues in quantum mechanics; see Putnam [1968]), and furthermore allows the exploration of new conceptual possibilities (for instance, new consequences about our understanding of quantum theory). So, the alternative provided allows one to make sense of science in a more straightforward way than Field’s approach does, given the existence of proposals suggesting the need for a change of logic on empirical grounds.

Two new features are introduced by this empiricism. The first of these is an account of logical change that goes hand in hand with the formulation of physical theories. In this way, Field’s objection against logical non-apriorism can be rebutted: with a piecemeal approach to theory construction in science, it is possible to change the underlying logic of a particular theory, without ever assuming that any such logic is empirically indefeasible. Depending on the context under consideration, a change in logic may well be warranted (more on this below). Second, since there is no requirement that the resulting theories be true, but only empirically adequate, the account advances a thoroughly anti-realist view about science (see van Fraassen 1980 and 1989). Thus, fallibilist empiricism provides a way of incorporating logic into an empiricist framework, without presupposing the analytic/synthetic distinction.

2. Field’s Arguments for the Apriority of Logic

Field puts forward three major arguments in defense of logical apriorism, and concludes his discussion with a challenge to those who deny that logic is a priori (in the sense that logic is empirically defeasible). I shall examine these arguments in turn.

2.1. The Irrelevance of Empirical Factors

Field’s first argument, which stresses the irrelevance of empirical factors to the revision of logic, is as follows. Even if there were empirical factors indicating that a logic should be revised, these factors are dispensable—that is, they only help us see a conceptual possibility that we should have been able to recognize independently. So, for instance, if some quantum mechanical phenomena suggest the revision of the distributivity principle on empirical grounds, we should be able to recognize the possibility of this revision independently of the phenomena. Similarly (although Field does not think that this example is decisive (Field 1998,
Is Logic A Priori?

p. 3)), if the discovery that there is no Santa Claus suggests that we need to change classical logic to accommodate non-referring terms, we should have been able to recognize this possibility on purely conceptual grounds. Thus, the move to a version of free logic—to take into account terms that lack reference—could have been made based on conceptual considerations alone. In other words, according to Field, we should not say that logic is empirically defeasible: empirical phenomena only suggest to us conceptual possibilities.

What are the problems with this argument? In a nutshell, it cuts too much. If the argument is right, it could be used to establish that science is a priori. After all, instead of saying that a particular phenomenon falsifies a given theory, we could say that the phenomenon only indicates a possibility that we should have been able to recognize independently. If we only consider the structure of Field’s argument, what is the difference between the presumed empirical indefeasibility of logic and the empirical defeasibility of a scientific theory? Of course, it simply begs the question to claim that logic is a priori but a scientific theory is not. But then, it is unclear what is supposed to distinguish a scientific theory from a logic in this context. In other words, if Field’s argument is sound, it could be used to establish that even scientific theories are empirically indefeasible. The argument had better not be sound then.

The problem is that although an empirical phenomenon may suggest a conceptual possibility, it does not follow that the change suggested by the phenomenon in our theory is purely conceptual. Since the phenomenon is actual, it suggests a change of an empirical kind. Field’s argument seems to disregard this fact. But this is exactly the same type of fact that would motivate a change of logic on empirical grounds. It is only by assuming that logic is a priori—as opposed to scientific theories, which are not—that Field could establish that his argument does not also make scientific theories empirically indefeasible. But this begs the question.

2.2. The Ubiquity of Logic

This last remark leads us to Field’s second argument. Field asks what could possibly count as the empirical defeasibility of a logical principle. In order to “refute” a logical principle, we need a logic, and such a logic has to be a priori; otherwise, it would be as open to refutation as the logical principle under discussion. This argument also has a more general form. We need a logic both to draw consequences from an evidential system and to evaluate between rival systems. And if logic were not taken to be a priori, we would not be able to perform these roles, since the logic itself would then be open to revision along with the evidential systems under consideration.\(^2\)

This is a plausible argument, but it is far from being decisive. Every evidential system does have an underlying logic. But why must it be a priori? The logic could simply be accepted as adequate in that context, being used for the purposes of drawing empirical consequences. This does not mean that it can not be revised even on empirical grounds. The decision not to revise a logic in a given context is pragmatic, since we need to use a logic to draw consequences. But there is nothing in this process that requires logic to be a priori. The only requirement is that the logic is accepted in that context.
We may even have good reasons to change the underlying logic of our evidential system. For example, if we want to accommodate the constructive features of mathematical reasoning, classical logic will be clearly inadequate. In this context, an intuitionistic logic would be far better. Similarly, if we want to accommodate the existence of inconsistencies in our evidential systems—without arbitrarily rejecting some bits of information—we have reason to move to a paraconsistent logic, instead of adopting classical or intuitionistic logic. In other words, there are logics adequate to some contexts, and inadequate to others.

The adequacy of a logic to a particular context, or domain, depends on three factors: (a) structural features of the logic (the kind of inferences it licenses); (b) features of the domain under consideration (if the domain incorporates inconsistencies, say), and (c) the particular uses we want to make of the logic (our aims in that particular context). For example, intuitionistic logic typically does not allow the use of excluded middle in reasoning about infinite sets (factor (a)). This logic then becomes a crucial tool for those concerned with the study of constructive features of mathematical thought (factor (c)), especially because so much mathematics depends on constructions involving infinite sets (factor (b)). Similarly, a paraconsistent logic typically does not allow the derivation of any sentence whatsoever from an inconsistency (factor (a)). So, if we are dealing with an inconsistent theory (factor (b)), and we do not want to lose information (factor (c)), we have reason to move to a paraconsistent logic. This logic is adequate to this context.

Once we realize that it is the interplay of these three factors that determines the adequacy of a logic to a given context, it becomes clear that there is no reason why logic should be taken as a priori. A logic can be used to draw consequences from an evidential system despite being empirically defeasible. It only has to be accepted in that context.

So, the fact that a logic is always presupposed by an evidential system does not entail its apriority. This logic could still be revised on empirical grounds. Of course, the possibility of such a revision should be more than epistemic; that is, it has to be a genuine possibility. But how could this possibility be genuine? A logic can be taken as inadequate if it licenses consequences which do not reflect the structure of the domain under consideration—that is the whole point of stressing the interplay of factors (a)-(c). The idea is that a logic is associated with structures which reflect the information about the relevant domain. For example, in quantum mechanics, the logic of elementary statements reflects the structure of the Hilbert space used to represent the states of a quantum system (for details, see, for example, van Fraassen 1991 and Rédei 1998). The logic of elementary statements is not classical, and classical logic does not generate the right structure in that domain. So, even if we start with an evidential system that has a particular logic (say, classical logic), there may well be reasons to change this logic (moving, for example, to quantum logic). In this way, a logic may be empirically inadequate, in the sense of not being adequate to the domain to which it is applied.

In other words, despite being presupposed by an evidential system, a logic may turn out to be inadequate. This fact provides a reason for us to change the logic on empirical grounds—or, at least, on grounds that depend on the domain.
under consideration. In this way, there is no incompatibility between a logic being presupposed by an evidential system and the genuine possibility of its empirical revision. The revision of a logic, of course, need not always be empirical—some cases of logical change are clearly motivated by extra-logical considerations. And this is enough for the logical non-apriorist, who need not be committed to the claim that all changes in logic are empirical, but only that some are. Ultimately, as long as changes in logic do not depend on purely logical considerations—and some are indeed empirical—the logical apriorist is on safe grounds. After all, being domain-dependent, such changes would be motivated by considerations that are not only of a logical sort. And so, extra-logical considerations play a crucial role (for further elaboration on this point, see Bueno and Colyvan 2004).

In summary, there is no doubt that we need a logic to formulate any theory about the empirical world. Moreover, if the logic is changed, the theory is also changed. But this does not entail that logic is a priori. As we saw, there may well be empirical reasons to revise the logic, on the basis, say, of its inadequacy to the domain in question. Furthermore, the need for a logic to formulate a theory does not entail that the logic cannot be revised either. Even if the theory changes with any change in the logic, this does not mean that there is something privileged about the first combination of theory and logic. That particular combination is often the result of a contingent process, the result of a historical accident. For example, what we call classical mechanics is the combination of (roughly speaking) a number of physical principles from mechanics with classical logic—or, alternatively, a certain family of models formulated in a language whose logic is classical. So, if we changed that logic, we would have ipso facto changed the theory. But, of course, classical mechanics is the result of a long, and largely contingent, historical process. The original combination of physical principles and logic that constituted classical mechanics might have been very different: the logic in question might have been non-classical. So, the fact that we need a logic to formulate a theory does not establish much about the nature of that logic; in particular, it does not establish the logic’s apriority.

But it might be argued that there is a further difficulty here. In order to formulate a theory in the first place, we need a logic. If that logic is deemed inadequate to the domain, what kind of logic is used to determine the relationship between the domain and the logic in the first place? In answering this question, we seem to face a dilemma: either the logic that allows us to conclude the empirical inadequacy of the original logic is fixed, or it is not. If the logic is fixed, we are back to the apriority of logic—after all, if the logic is not open to revision, it is not open to empirical revision. If the logic is not fixed, a regress seems to threaten—after all, we would need a further logic to assess the adequacy of the second logic, and so on. Is there a way out?

I think there is. With regard to the first horn, it does not follow that if the logic used to establish the empirical inadequacy of the original logic is fixed then the second logic is a priori. The logic may be fixed in that particular context, but there may well be reasons to revise it—even on empirical grounds—in other contexts. In particular, note that the context in which a logic is assessed is a different context from that in which the logic is used to draw consequences. And there is no reason to assume that the logic should be fixed throughout.
But if the logic is not fixed in general, does a regress not threaten? (This is, of course, the second horn.) I do not think it does. For fallibilists, a regress is never a problem. Logical fallibilists are not in the business of establishing the One True Logic about the world. In order to determine the adequacy or inadequacy of that first logic, we can choose a logic on pragmatic grounds, and then proceed the evaluation of the first logic in terms of the second. As with any judgment, judgments about the adequacy of a logic are always fallible, and we may need to revise them. But this does not mean that we do not have good (albeit fallible) reasons to make them. So, in either case, the dilemma can be undercut. (I will return to this issue in section 3, below.)

It might be objected that we can always keep a logic by suitably changing the mathematical or physical principles used in the formulation of a given scientific theory. For example, in quantum mechanics, we could keep classical logic but accept non-standard probabilities; or perhaps we could adopt an interpretation of the formalism that allows us to maintain a thoroughly classical view combined with a non-classical “internal” logic (see van Fraassen 1991). The picture here is akin to the Duhem-Quine situation, where the One True Logic can always be maintained, should we decide to do so, as long as changes are made elsewhere in our overall belief system. This is, of course, weaker than the position discussed above—the argument is not that logic is indefeasible, but that it never has to be defeated.

This is certainly a possibility. But if it is right, in what sense can we say that logic is a priori? In this Quinean picture, the “indefeasibility” of logic becomes the result of a methodological decision; it has nothing to do with any structural feature of logic itself. Basically, it is a matter of deciding which parts of the belief system we are going to revise. One of the outcomes of the Quinean picture is the naturalization of all the components of the web of belief. As a result, even logic can be empirically defeated if we are unwilling to make changes elsewhere in the system. But then it is not at all clear in what sense logic is a priori.

But there is a further reason why the presupposition of a logic by any evidential system is not enough to establish that it is a priori. For there might be several logics appropriate for the same domain. For example, as is well known, there are infinitely many paraconsistent logics (see, for example, da Costa 1974 and 1997, and da Costa, Krause, and Bueno 2007). Thus, if we want to tame the inconsistency of a given domain, there will be several paraconsistent logics appropriate for this task. Should we claim that all such logics are a priori? That does not seem plausible. First, these paraconsistent logics are not equivalent. They form a hierarchy of successively weaker logics, in the sense that a contradiction that trivializes one logic in the sequence will not trivialize logics that come later in the sequence (see da Costa 1974, and da Costa, Krause, and Bueno 2007). As a result, if one of these logics can be used to accommodate an inconsistency in the domain under consideration, then all weaker logics in the sequence also can. But (and this is important) not all logics in the sequence will be adequate to accommodate the inconsistency, given that some will be trivialized by the latter. Of course, this will depend on the particular type of inconsistency under consideration (the domain in question) and the particular paraconsistent logic in use (especially, the structural features of the latter). As a result, given that such
Is Logic A Priori?

paraconsistent logics can be revised on empirical grounds—given the features of the domain in question—it is unclear in what way these logics can be a priori.

Thus, prima facie at least, it seems difficult to reconcile the thesis of the apriority of logic with logical pluralism (that is, the claim that there are several logics adequate to a given domain). As noted in the introduction, logical apriorism is usually associated with a dogmatic attitude about logic, and it does not seem to be coherent with a fallibilist approach. Similarly, logical apriorism sits more comfortably with a monist view of logic (that is, with the claim that there is only one logic adequate to the world). After all, if logic is not empirically defeasible, there seems to be no principled way for the logical monist to explain the existence of different applied logics (in particular, of different logics applied to the vernacular). For according to the logical apriorist, empirical considerations are irrelevant to the choice of alternative logics. In this way, the logical apriorist seems to rule out the plurality of applied logics. Given that the latter is a fact of contemporary logical theorizing, I take it that logical apriorism is ultimately inadequate to make sense of logic.

The picture that emerges here is, of course, of logical pluralism and empiricism about logic, and I will return to it in the last section of the paper.

2.3. Logic and Geometry

Field’s third argument is put forward to dispel the following worry that arises once one defends the apriority of logic. If the information about a given domain holds independently of experience, is this not a reason to doubt the adequacy of that information? In other words, the independence of experience seems to cast doubt on the acceptability of the information in question. Similarly, in the case of logic, if the latter is taken to be independent of experience, does this not establish the inadequacy of logic? As is well known, for centuries, geometry has been taken to be a priori—until it was shown that Euclidean geometry was inadequate to describe the world. Cannot the same happen with logic?

Field does not think so. In the case of our fundamental logic, it simply makes no sense to allow it to be empirically defeasible, since we need logic to formulate any theory about the empirical world. However, as he points out, geometry has never played this role in our world picture.

The problem with Field’s reply is that it disregards an important similarity between logic and geometry: both can be studied either as pure systems or as applied ones (see da Costa 1997). A pure geometry is the study of certain geometrical spaces on purely mathematical grounds. For example, when one investigates the mathematical properties of Euclidean spaces, of non-Euclidean spaces and so on, what is under investigation is a pure geometry. An applied geometry arises from the use of such geometrical spaces to describe the structure of the physical world. This is beautifully illustrated with the formulation of spacetime theories. The crucial point is that the geometry that is revised on empirical grounds is not pure geometry, since the latter is studied as a pure mathematical theory—it is applied geometry that is so revised.

Similarly, in the case of logic, we should distinguish between pure and applied logic (da Costa 1997). A pure logic is roughly the abstract study of consequence relations in a given language, independently of the nature of the
objects under consideration. When a logic is formulated abstractly, its semantics characterized and, for example, a completeness result established, what is studied is pure logic. An applied logic arises with the use of a logic to draw consequences about a particular domain. For example, when a logic is employed to derive consequences from a physical theory, it is taken as applied logic.\(^7\)

Just as with applied geometry, we may have more reason to revise a logic, in a given domain, than to change the principles of a physical theory. If we change the logic, and keep the physical principles, we may end up with a theory that is more empirically adequate and simpler than the extant alternative. Quantum mechanics provides an interesting example in this regard. A number of difficulties faced by the theory can be solved if we replace classical logic by quantum logic and revise the distributivity law of the former (see Birkhoff and von Neumann 1936, Putnam 1968, and Dickson 2001). The principles of quantum mechanics plus classical logic generate consequences that are either false or meaningless, from the viewpoint of some interpretations of the quantum formalism (see da Costa 1997, and Bueno and Colyvan 2004). This suggests that changing the underlying logic to a quantum one may provide a better picture.\(^8\)

But what is the impact of the distinction between pure and applied logics on the two components of the apriority of logic? Recall that the components are: (i) the legitimacy of logical principles does not depend on experience, and (ii) logical principles are empirically indefeasible. It is uncontroversial about the apriority of logic that (i) is true. What is problematic about logical apriorism is not the legitimacy of logical principles, which can be determined by the investigation of their logical form alone. The problem emerges when we consider the revisability of these principles on empirical grounds, that is, when we examine (ii).

Not surprisingly then, if we consider a pure logic, (i) is trivially true, and (ii) does not even arise, since a pure logic is not about any empirical domain, and so it cannot be revised on empirical grounds. The same happens with a pure geometry: one cannot refute the properties of a Euclidean space on empirical grounds, since no term in the theory of Euclidean spaces refers to an empirical domain. Therefore, no empirical refutation of the theory is possible. A pure logic also lacks terms that refer to the empirical world, and so any such refutation is impossible.

The picture changes when we move to applied logic. With regard to (i), the legitimacy of a logical principle is still independent of experience, for what matters here are considerations about logical form alone. But we cannot say the same with regard to (ii). In order to apply a logic, we have to provide an interpretation of the formalism; that is, we have to assign parts of the formalism to the world. The process is straightforward with regard to the individual terms and predicates of the language. All we have to do is provide an interpretation, in the standard way, defining a domain \(D\) of interpretation and assigning elements of \(D\) to the individual terms and \(n\)-tuples of such objects to \(n\)-place predicates. But what should be said about the logical constants? Roughly speaking, they are concerned with combinations of truth-values of the sentences (or formulae) of the language. However, depending on the domain under consideration, some of these combinations may be inadequate. The combinations may not describe the relations found in the relevant empirical domain. For example, in the case of quantum
mechanics, the distributivity law of classical logic fails to describe the relation between the results of certain empirical observations (namely, the structure of elementary statements). This suggests a way in which logical principles can be revised, namely, if they are inadequate to a given empirical domain. Moreover, this also provides an answer to the question raised by Field with his second argument: what could possibly count as an empirical refutation of a logical principle? In a nutshell, an empirical refutation emerges from the inadequacy of a logic to describe the structure which emerges in the relevant empirical domain.

In other words, if we consider applied logics, as opposed to their pure counterparts, the possibility of revising logical principles on empirical grounds becomes as clear as the possibility of revising geometrical principles. Thus, at the applied level, there is a close similarity between logic and geometry. At this level, none of them is so fundamental as not to be empirically defeasible. If these considerations are nearly right, then Field has failed to establish that logic is a priori, and a case can be made to support logical non-apriorism (the view that applied logical principles can be revised on empirical grounds). Field concludes his discussion by issuing the challenge from which this non-apriorism derives: how we can make sense of a logical principle being revised on empirical grounds? How can an evidential system incorporate a logic that is not a priori? If we are unable to answer these questions, we will not be able to articulate a non-apriorist view about logic. I think the best way to address this challenge is to indicate an account of logic that accommodates the issues raised by Field. What follows is just a first step toward addressing the challenge, and I will do that, in part, by systematizing some claims I have already made.

3. Logic: An Empiricist View

The account of logic suggested here has three major features: (1) logical pluralism, (2) context dependence, and (3) non-apriorism. The three features are closely related. I shall consider them in turn.9

(1) Logical pluralism is the view according to which there are several applied logics adequate to a given domain.10 Given a finite sample of reasoning \( S \), there are several logics adequate to \( S \) (in the sense that they appropriately describe the consequence relation exemplified in \( S \)). So, typically, if a logic is adequate to a given domain, several others will also be. To illustrate this: an inference which does not involve negation and is valid according to classical logic will also be valid according to any of the paraconsistent logics which characterize da Costa’s C-logics (da Costa 1974, and da Costa, Krause, and Bueno 2007). As noted above, the C-logics are a hierarchy \( C_n \), \( 1 \leq n \leq \omega \), of paraconsistent logics, such that the logic \( C_1 \) is strictly stronger than \( C_{n+1} \), in the sense that a contradiction that trivializes the former does not trivialize the latter. Moreover, each of these logics incorporates as much of classical logic as possible given their paraconsistent nature (\( C_0 \) is classical logic). So there is a sense in which classical logic is contained in da Costa’s paraconsistent logics. In this way, if the reasoning \( S \) considered above does not involve negation, there will be infinitely many paraconsistent logics which will be adequate for it, in the sense that the consequence relation found in \( S \) is licensed by such logics. Cases such as this clearly support logical pluralism.11

(2) That logic depends on the domain (or the context) to which it is applied
becomes clear when we consider applied logics. Depending on the domain under consideration, a logic might be clearly inadequate to be applied to that domain. To mention some examples again: as a representation of the constructive features of mathematical reasoning, classical logic is evidently inadequate.\textsuperscript{12} Classical logic is also inadequate to accommodate inconsistencies in a domain without losing information about it. On the other hand, quantum logic seems to be \textit{adequate} to the quantum domain, since it adequately represents the consequence relation between elementary statements of quantum theory, and preserves the extant information about the domain (where the distributivity law of classical logic seems to fail).\textsuperscript{13}

The fact that a particular logic is tied to a certain domain also introduces factual conditions. Depending on the domain under consideration, we may have to change the logic to formulate adequately the structures describing that domain. The development of quantum logic also illustrates this point. As is well known, classical propositional logic is associated with a Boolean algebra. Now, with the work of von Neumann and Birkhoff in the foundations of quantum mechanics, it has become clear that the lattices associated with quantum systems are not Boolean, since they are not distributive. A different kind of structure was required to describe these structures: modular orthocomplemented lattices (for a discussion, see Rédei 1998).

(3) The \textit{non-apriority} of logic now emerges as the natural outcome of the two previous features of logic. As we saw, the adequacy of an applied logic depends on the domain under consideration (the logic can be revised on empirical grounds), and there might be several logics adequate to that domain. As a result, a logic may be empirically revised if it turns out to be inadequate to the domain in question.

But this remark raises a difficulty: In order to show that a logic is inadequate, we need a logic. What is the status of the latter? Is such a logic a priori? (Of course, this is a version of one of Field’s argument for the apriority of logic.)

There is something right in this worry: we certainly need a logic to evaluate the adequacy of any given logic. However, this does not establish that the logic used in the evaluation is (or has to be) a priori. As already noted, all we need is to \textit{accept} that logic. The latter can even be revised in a different context. We have here a kind of “bootstrap” strategy in which a logic, in a given context, is used to evaluate another logic. In fact, we evaluate a \textit{package of a logic plus a theory} about the domain in question against another \textit{package} where the underlying logic used in the first package is changed. And as with any case of theory choice, we decide among the rival packages based on \textit{pragmatic} factors, such as simplicity, informativeness, interest, problem-solving effectiveness, et cetera. The evaluation is carried out, of course, using a particular logic. The only requirement is that such a logic be \textit{accepted} in that context.

But how should we decide which logic to use in the evaluation of alternative packages? I do not think there is an a priori answer to this question. (That is the whole point of the present approach!)\textsuperscript{14} But note that even in the case of quantum mechanics, classical logic can be used at the meta-theory to evaluate the two well-known packages: one that uses classical logic, one that adopts quantum logic. And it does not follow that the adoption of classical logic at the meta-theory immediately provides support for the classical logic approach. Usually, quantum
logicians argue for their proposals using classical logic at the meta-theory: there is no incoherence here, nor any support for the classical logic view. After all, the domain under consideration at the meta-theory is different from that of the object theory: the latter deals with quantum phenomena, the former with theories about such phenomena. Quantum phenomena may require theories for which non-distributivity holds, but the choice of theories about quantum phenomena may not involve any failure of distributivity. Of course, I am not claiming that classical logic should be used at the meta-theory in the evaluation of alternative packages. My only point is to indicate that its use is not enough to favor the classical logic alternative. In this way, we can see that logic does not have to be a priori for it to play the roles we need it to play.

Since the present proposal is fallibilist, an infinite regress of meta-theories with different logics is not threatening. The point is not to establish that there is one, and only one, logic adequate to a given domain. Given logical pluralism, there might be several such logics. But the regress can be broken using pragmatic considerations, evaluating the resulting packages in terms of their simplicity, explanatory power, problem-solving effectiveness, et cetera.

But then does pluralism not degenerate into relativism? Does it not become arbitrary, a matter of whim, which logic one uses? I do not think so. There are still criteria by which to choose between alternative logics. A logic should be adequate to the domain to which it is applied. And the adequacy depends on the three factors discussed in section 2.2 above (the structural features of the logic, the features of the domain in question, and the uses we want to make of that logic). With these criteria, we are able to exclude some logics to perform some tasks—but not all of them. We still have pluralism, but no relativism.

4. Conclusion

Far more could be said about the issues addressed in this paper. But I hope I said enough to sketch an answer to Field’s challenge (mentioned at the end of section 2.3). First, we can make sense of a logical principle being revised on empirical grounds, if that principle is inadequate to the domain to which it is applied. And the adequacy of a logical principle is a function of the three factors just mentioned. Second, an evidential system can incorporate a logic that is not a priori, provided this logic is adequate to the domain under consideration (in the above sense), and that the logic is accepted in that context.

A concluding word. It should be clear by now that the present approach is inspired by the writings of one of the greatest empiricists about logic in the 20th century: John von Neumann. In an unpublished manuscript, written in 1937, shortly after the publication of his paper on quantum logic with Birkhoff, von Neumann observed:

The basic idea is that the system of logics which one uses should be derived from aggregate experiences relative to the main application which one wishes to make . . . logics should be inspired by experience. (von Neumann 1937, p. 2)

Quite so. φ
Notes

1 My thanks go to Jean-Yves Béziau, Mark Colyvan, Newton da Costa, Steven French, Hugh Lacey and David Miller for helpful discussions and suggestions. Several points examined here have arisen from discussions with Newton da Costa. Steven French has also made a number of particularly useful comments on an earlier version of this work, which led to substantial improvements.

2 An evidential system is a pattern to formulate and update our beliefs. As Field points out (Field 1996 and 1998), logic is presupposed by every evidential system, since we must use a logic to articulate and revise our beliefs.

3 As noted above, Field doesn’t think that epistemic modality is a genuine modality.

4 Although this kind of empiricism is clear in “Two Dogmas of Empiricism” (Quine 1953), Quine himself adopted a more conservative view about logic in his philosophy of logic book (Quine 1970).

5 A contradiction trivializes a logic if everything follows from such contradiction given that logic. As is well known, classical logic is trivialized by any contradiction. This is not the case, however, for paraconsistent logics, which in general are not trivialized by at least some contradictions (see da Costa, Krause, and Bueno 2007).

6 I have more to say about applied logic in the next section.

7 As with so many other demarcations, the distinction between pure and applied logics is vague. What matters, however, is that we have clear examples of logics which fall under each category (so it’s not a trivial demarcation). Despite the vagueness, the distinction is still useful, as I shall indicate in what follows.

8 Of course, I’m not suggesting that this establishes the truth of quantum logic. The point here is only to motivate the possibility of an empirical revision of logic, similarly to what happened in the case of geometry.

9 The proposal suggested here was inspired by accounts of logic found in the works of da Costa, van Fraassen and von Neumann (see, for instance, da Costa 1997, van Fraassen 1971, von Neumann 1937, and Birkhoff and von Neumann 1936). But probably none of them would fully endorse all of the components combined here.

10 As noted above, in order to investigate a domain, we typically need a logic. In this sense, logic is presupposed in such investigation. The point of logical pluralism is to stress that more than one logic may be adequate for this task.

11 For further discussion of logical pluralism, see da Costa 1997, Bueno 2002, Beall and Restall 2006, and Bueno and Shalkowski 2009—all of which provide defenses of different forms of logical pluralism. Logical monism is defended in Priest 2006.

12 The domain here consists in the constructive features of mathematical reasoning. Even a classical logician should acknowledge the limitation of classical logic here, since it licenses a number of non-constructive inferences.

13 An important reason supporting the claim that logic is domain-dependent is that we can make sense of logic in a better way: its role in theory construction in science can be better appreciated, and how it informs our understanding of the empirical world can be properly evaluated. This motivated von Neumann to defend logic’s domain-dependence in several works (see, for example, von Neumann 1937, and Birkhoff and von Neumann 1936).

14 The logical apriorist presumably will argue that we need to use an a priori logic to choose between alternative packages. In fact, the apriorist will probably deny that we will ever be in a position of having to choose between such packages (of alternative logics). But this raises a difficulty for apriorism. As noted above, it is difficult to see how the logical apriorist can make sense of logical pluralism.

References

Is Logic A Priori? 117


