The *Manual* of Domninus

By Peter Brown

I present an English translation of the Manual of Elementary Arithmetic *by the fifth-century philosopher Domninus, along with an introduction, some brief notes on the Greek text of Boissonade, and some comparisons with Euclid, Nicomachus, and Theon.*

Introduction

The spectacular success of Greek geometry stands in stark contrast to the relative obscurity of Greek arithmetical theory. While school children are (or were) brought up on the first four books of Euclid's *Elements*, very few have read the arithmetical books that follow. Although Greek arithmetical theory has been little known even by mathematicians and classicists, some concepts in the works of the arithmeticians have survived the centuries. The notions of even and odd, cube and square, and perfect, deficient and abundant numbers are still used today.

Those familiar with Greek arithmetic theory would probably have obtained that familiarity through Nicomachus's *Introduction to Arithmetic* (c. 100 C.E.), but it is of some importance to appreciate how the material was viewed in later antiquity. Indeed, the elements of mathematical mysticism and numerology, which were entirely absent from the work of Euclid, became increasingly important in Neoplatonic philosophy, especially in Iamblichus and Theon, and Domninus's Manual provides a refreshing return to a rationalist approach to mathematics. The only translation of this arithmetical work is in French, the work of Paul Tannery in 1906,[1] and so I believe that a modern English translation of the work is warranted.

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The survival of the *Manual* is no doubt due to its pedagogical excellence. The basic principles of Greek arithmetic theory are presented very clearly with simple explanations of the main ideas and concepts, and Domninus does not allow himself to go off the track. Moreover, the metaphysical aspects of number and their associated philosophical difficulties are completely ignored, making the work one of mathematics rather than philosophical speculation.

Domninus belongs to the fifth century C.E. and originated from the town of Larissa in Syria. He and his contemporary Proclus were students of Syrianus, the head of the Neoplatonic school at Athens; it appears that Domninus and Proclus may have become rivals. Some biographical information remains.

a. A short account from the lost *Onomatologos* of Hesychius, as preserved by Eudocia: "Domninus, philosopher, Syrian by birth, from Laodiceia and Larissa, a town in Syria, a disciple of Syrianus and schoolmate of Proclus, as Damascius says. He wrote against the opinions of Plato."[2]

b. A passage from the *Life of Proclus* by Marinus of Samaria that lends strength to the conjecture that there was a degree of rivalry between Domninus and Proclus: "Syrianus had indeed planned to explain to him (Proclus) and to the Syrian Domninus, philosopher and successor (*diadochos*), one of these works, the Orphic writings or the [Chaldean] Oracles and had left the choice to them. But they did not agree and did not choose the same work, Domninus choosing the Orphic, Proclus the Chaldean. This disagreement hindered Syrianus from doing anything, and the great man soon died."[3] It appears in this passage that Domninus, not Proclus, became the immediate successor of Syrianus as head of the school at Athens. It may be the case that the phrase "philosopher and successor" was originally an explanatory gloss that became part of the text, since Marinus elsewhere refers to Proclus as "successor."[4] The question still remains as to where this gloss originated and, more importantly, whether there is any truth in it. Even accepting this (high) probability of interpolation, the possibility remains that Domninus became "successor," left Athens at some stage shortly thereafter, perhaps under duress, and the position of *diadochos* passed to Proclus.

The biographer Marinus is clearly a very enthusiastic supporter of Proclus, as some of his comments in the early section of the biography evidence; he could well be expected to leave out any mention of Domninus as "successor." Moreover, Marinus refers to the succession as the "Golden Chain" of philosophers, which he seems to have regarded as divinely predetermined. If Domninus did become the successor of Syrianus for a short time and was later removed from the position under acrimonious circumstances, Marinus and others whose allegiances were to Proclus could have regarded the succession as a mistake and consequently have chosen to ignore its occurrence.

The reason for the friction between Domninus and Proclus seems to have been the former's "heretical" (i.e. anti-Platonic) views, referred to above. The distinction between the physical and metaphysical aspects of number, which appears to have been very important for

Syrianus, Iamblichus, and also Proclus, is glaringly absent in the *Manual*. Also, Proclus thought very highly of Nicomachus's work, believing (having been so informed in a dream) that he possessed the soul of Nicomachus.[5] Further, according to Suidas, Proclus wrote a commentary on Nicomachus's *Introduction to Arithmetic*. Domninus, on the other hand, undermines and often rejects much of Nicomachus's arithmetical theory.

c. A longer extract from the *Life of Isidorus* by Damascius, which Suidas preserves: Domninus was "a man capable in mathematics but superficial in other philosophical subjects; also, he often shaped the opinions of Plato to his own and corrupted them by the colour that he gave them. He was, however, corrected by Proclus, who wrote against him a 'purifying treatise' [that was the title] of the teaching of Plato. His way of living was not at all accomplished and does not merit to be truly called that of a philosopher; thus, the oracle of Asclepius at Athens prescribed the same remedy to the Athenian Plutarch and the Syrian Domninus.... The remedy consisted of gorging himself on pork flesh.... It was said that if [Domninus] went for one day without taking some, his sickness immediately seized him until he was once again satiated with the meat. Asclepiodotus, while still very young, saw him as an old man. He seemed to have an affected air of superiority and of severity, hardly deigning to speak to individuals or to strangers who saw him, even those with claims to a certain distinction. In any case, Asclepiodotus found his behavior quite offensive. In regard to a certain topic in arithmetical theory, he did not think he ought to submit to Domninus's views purely on the grounds of his youth, neither to be put off by some mild objection; rather, he tried to refute Domninus's argument, though the latter ceased to acknowledge his conversation or comment." This extract confirms the general dislike of Domninus by later members of the school and again emphasizes Domninus's shift away from the Neoplatonic tradition, which led to his vilification. Interestingly, the writer is prepared to acknowledge Domninus's mathematical ability.

Domninus's *Manual of Introductory Arithmetic* is clearly aimed at the would-be philosopher rather than the would-be mathematician, with no proofs or even justifications of any results being given. The work is designed to summarize and, more importantly, simplify the elements of arithmetic that had been previously presented by such writers as Nicomachus, Iamblichus, Plato, Theon, and Euclid; as such, it is very successful. The chief sources seem to be Nicomachus, Euclid (*Elements* Bk. 7), and a third unknown source that was also used by Iamblichus. Presumably, this third source contained material on the four "classes" of numbers, since this topic is absent from the other two sources (see Chapters 14-17). Domninus's basic source of knowledge is Euclid: he quotes Definition 1 of Book 7 of the *Elements* in the first line of the *Manual*.

It must be pointed out, however, that Domninus does not slavishly follow Euclid at every point and often presents non-Euclidean definitions. This point needs to be emphasized, since several modern historians state that Domninus's work represents a return to Euclidean arithmetic theory in preference to that of Nicomachus.[7] Tannery himself writes, "Son *Manuel* offre cependant un certain intérêt, surtout en ce qu'il témoigne d'une tentative sérieuse de réaction contre Nicomaque et de retour à Euclide." While there is much truth in this, it must be recognized also that Domninus seems to seek some middle ground between the unnecessarily complex classification of Nicomachus and the simple brevity of Euclid.

Domninus attempts to be original through synthesis, not merely antiquarian. Although Domninus does not mention Nicomachus by name (he refers to the "ancients" at various places, among whom Nicomachus is presumably meant to be included), he clearly undermines the often very complicated nomenclature that Nicomachus had introduced into arithmetical theory. For example, Nicomachus's classification of odd numbers into three types—the primes, the odd composites, and, a rather artificial and confusing third subdivision, the numbers that may be composite in themselves but are relatively prime to other numbers—is reduced to just the first two classes, and the third phenomenon is relegated to a different section of the *Manual.*

Simplification is very common in Domninus, although he does not prune the classification all the way back to Euclid. For example, he discusses the division of numbers into perfect, abundant, and deficient, whereas Euclid mentions only the first type. It is important to observe that, as well as leaving out much of the useless baggage from Nicomachus, Domninus has retained a mention of the quaint terms "columns," "tiles," and "altars," which imply distinctions that are no longer regarded as worth making (see Section 5). Domninus has also left out any reference to figurate numbers, which were, and are in fact, useful and interesting to mathematicians. (This may be because the section in Nicomachus dealing with figurate numbers appears in Book II, and Domninus may have only intended his handbook as a reworking of the material in Book I.)

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There is no new mathematics in Domninus's work. Even the result that the geometric mean of two numbers is also the geometric mean of the arithmetic and harmonic means of those numbers is probably not due to him, although it does not appear in any of the earlier extant writers. Nonetheless, Domninus does introduce a successful novelty into his work. He considers numbers in relation to themselves and numbers in relation to one another separately, notions that often seem to be confused in Nicomachus. By distinguishing these two concepts, Domninus is able to explain more clearly the elementary subdivisions of numbers found in the earlier writers.

The Greek text I have used is that of Boissonade,[8] which contains many errors. Unfortunately, I have not had the opportunity to examine the manuscripts on which Boissonade's text is based. Details of the manuscript tradition can be found in Tannery's "Le Manuel" and his "Notes Critiques,"[9] on which I have relied heavily. In addition to the *Manual,* there is another work regarded as being from the pen of Domninus (or at worst written at the same time), *How to Take a Ratio From a Ratio.* This work, with critical notes, has been translated into English by Jacob Klein.[10]

I have attempted to be fairly literal in my translation, while making the English as readable as possible. At almost every stage I have followed the suggested changes of Tannery's "Notes Critiques," which are recorded in my notes after the translation, along with other alterations and readings that I propose. Minor errors such as the omission of an iota in the word *arithmos* in one place and the occasional omission of an accent, I have not bothered noting.

A Manual of Introductory Arithmetic by Domninus of Larissa

Section 1:

1. The **monad** is that by virtue of which each of the things that exists is called one.

2. A number is a collection of monads. The whole realm of numbers is a progression from the monad, by a difference of one monad, to infinity. There is the **monad**, then the **duad**, then the **triad** and **tetrad**, and so on in turn.

3. Some numbers are capable of being divided into two equal parts, such as 4 and 6. Others are not capable of undergoing this division, such as 3, 5, 7, and 9. None of these numbers is capable of division into two equal parts, because the monad is, by its very nature, indivisible.

4. Those numbers, therefore, that are divisible into two equal parts are called **even**, while those not capable of undergoing this division are called **odd**.

5. The even numbers start from the duad and proceed by a difference of two to infinity. The odd numbers start from the triad and proceed in steps of two to infinity.

6. Some even numbers can be divided and subdivided into two parts, always both equal and even, until the divisions reach the duad. For others this is not the case.

7. Those numbers for which repeated division yields the duad are called **evenly-even**, such as 4, 8, and 16.

8. Those (even) numbers not capable of undergoing this process are called **evenly-odd** and **oddly-even**, such as 6, 10, and 12.

9. Those numbers, then, that are evenly-even are each double the previous one, beginning from the duad and proceeding to infinity. The evenly-odd and oddly-even numbers also begin from the duad, which is the common beginning of all even numbers. They proceed by a difference of two to infinity, except of course for those

pairs in the midst of which falls some evenly-even number. For such pairs differ from one another by four, being two from either side of the evenly-even number between them.

10. Some odd numbers are capable of division into a certain number of equal parts, such as 9 into three triads, 15 into three pentads and also five triads, and 35 into five heptads and seven pentads. Other numbers, however, are not capable of division into any number of equal parts, such as 3, 5, 7, and 11.

11. Those (odd) numbers, then, that are capable of division into equal parts in any way whatsoever are called **oddly-odd**, while those not capable of undergoing this process are called **prime-and-incomposite** numbers.

12. Now, regarding the oddly-odd numbers, some are multiples of one odd number or of several odd numbers, but the prime-and-incomposite numbers are not multiples of any number. For this reason the duad is considered to be among the prime-and-incomposite numbers.

13. Well, now that we have considered the classification of numbers according to their form, let us examine the classification of numbers according to the multitude of monads that underlie them and constitute, as it were, the material of the numbers themselves.

14. Some numbers, then, appear in the units, some in the tens, some in the hundreds, and some in the thousands. Those numbers between one and nine appear in the units, those numbers ten times as great as those in the units appear in the tens, those ten times as great as those in the tens appear in the hundreds, and those ten times as great as those in the hundreds appear in the thousands.

15. So, these are the four classes of numbers, and every number belongs to one, to several, or even to all classes. 5 is found in the class of units; 25 is found in the class of units and the class of tens; 325 appears in the three classes, units, tens, and hundreds; 2325 appears in all the classes.

16. A similar theory holds for the myriads, for there are units of myriads, tens of myriads, hundreds of myriads, (thousands of myriads). They may be simple, double, or manifold, taking their name accordingly.

17. Furthermore, the simple myriad is the number 10,000 itself, the double myriad is a myriad times the simple myriad, the triple is a myriad times the double, the quadruple is a myriad times the triple, and so on forever.

Now it is part of logistical theory to speak further about these things, but, at this point of our discourse, we will make the remark that our entire examination of numbers thus far has been to consider them individually.

18. For any number whatsoever, when studied individually, is either even or odd in

regard to its form. If it is even, then it is either evenly-odd or oddly-even, but, if odd, then either oddly-odd or one of the prime-and-incomposite numbers. And it has been considered according to the underlying multitude of units in it: either it will be found in the units, the tens, the hundreds, or the thousands, or in several or all of these classes.

19. This, then, is our theory in regard to numbers in relation to themselves. It is also necessary to investigate their association with each other. Let us therefore start afresh from the beginning.

Section 2:
20. Some (pairs of) numbers have a single monad as their only common **factor**, such as 5 and 7, for there is no single number that divides them both. Others have one or many such common factors. In the first case we have 6 and 9, for they have only 3 as a common factor, while 6 and 12 have several common factors, for they are both divisible by 2 and 3.

21. Those numbers whose common factor is a single monad are said to be **prime to** each another, but those whose common factor is some number, either one number or many, are called numbers **composite to** each other.

22. This, then, is the theory in regard to numbers with respect to their association to each other by form, but the following must now be said about the relationship in regard to their underlying content, that is, in regard to the number of units that make them up.

23. Any number, when compared to any other number whatsoever in regard to its multitude of constituent units, is clearly either equal to it or unequal. If they should be equal to each other, their relationship to each other is unique and does not admit any other possible distinction, for the notion of equality is not able to be further subdivided, and things that are equal, are equal in only one and the same way.

24. Should (two) numbers, however, be unequal, one can consider ten possible relationships between them. It is necessary, however, for us to explain about the ratio of the (two) numbers and to make the remark that the smaller of two (unequal) numbers is either a part or several parts of the larger.

25. For if (the smaller) were to divide the larger, it would be a **part** of it, as 2 is of 4 and 6, being a half of the former and a third of the latter. But, if it were not to divide it, it would be a **proper fraction** of it, as 2, which does not divide 3, is two-thirds of it, and as 9 is in regard to 15, not dividing it, but being three-fifths of it.

26. This, then, having been established, we say that if two unequal numbers were placed before us for examination, the smaller either divides the larger or it does not. If it does divide, the larger is a **multiple** of the smaller and the smaller is a **submultiple** of the larger number, as is the case with 3 and 9. For 9 is a multiple of 3, being three times as great as it, and 3 is a submultiple of 9, being the subtriple of it.

27. If the smaller does not divide the larger, then, being subtracted from it either once or many times, it will leave a number somewhat less than itself, which will always be either a part or a proper fraction of it, for either it will leave a monad or some number. If, then, it leaves a monad, it is clear that it leaves a part of itself: the monad is a part of every number, since every number is a collection of monads. But if it leaves some number, that number will be a part or a proper fraction of it, since the smaller of two numbers is either a part or proper fraction of the larger.

28. If, then, having subtracted (the smaller) from the larger once, it leaves some number smaller than itself, which is a part of it, the larger will be a **superparticular** of the smaller and the smaller will be a **subsuperparticular** of the larger number, as is the case with 2 and 3. 3 is the superparticular of the number 2, for it is one and a half of it and for this reason is said to be the **sesquialter** (of 2), and 2 is the **subsesquialter** of 3. This is also the case with 6 and 8, since 8 is the **sesquitertian** of 6 and 6 is the **subsesquitertian** of 8.

29. Now, if the remaining number is a proper fraction of the smaller number, the larger will be a **superpartient** (of the smaller) and the smaller the **subsuperpartient** of the larger number, as is the case with 3 and 5. For 5 is a superpartient of the number 3, for it is one and two-thirds of it and for this reason is said to be the **superbitertian** of it, while 3 is the **subsuperbitertian** of 5. This is also the case with 15 and 24, for 24 is the **supertriquintian** of the number 15, being one and three-fifths of it, and 15 is the **subsupertriquintian** of 24.

30. If the smaller may be taken from the larger more often than once before leaving some number smaller than itself that is a part of it, then the larger will be the **multiple-superparticular** (of the smaller) and the smaller the **submultiple-superparticular** of the larger number, as is the case with 2 and 5. 5 is the multiple-superparticular of 2, for it is two and one-half of it, for which reason it is said to be the **duplex-sesquialter** (of 2) and 2 is the **subduplex-sesquialter** of 5. This is also the case for 6 and the number 26. 26 is the **quadruplex-sesquitertian** of the number 6 and 6 is the **subquadruplex-sesquitertian** of 26.

31. If the remaining number is a proper fraction of the smaller number, the larger is the **multiple-superpartient** (of the smaller) and the smaller is the **submultiple-superpartient** of the larger number, as is the case with 3 and 8. For 8 is the **duplex-superbitertian** of the number 3, and 3 is the **subduplex-superbitertian** of 8. This is also the case with 10 and 34, for 34 is the **triplex-superbiquintian** of the number 10 and 10 is the **subtriplex-superbiquintian** of 34.

32. These, then, are the so-called relationships between unequal numbers and also the names that the ancients gave them: the first, multiple; second, submultiple; third, superparticular; fourth, subsuperparticular; fifth, superpartient; sixth, subsuperpartient; seventh, multiple-superparticular; eighth, submultiple-superparticular; ninth, multiple-superpartient; tenth, submultiple-superpartient. This, then, is the theory of the reciprocal relationship of numbers in regard to their underlying mag-

nitude.

33. It seems good, then, to make an end here (of this section), since we have closely reviewed the whole of numbers in relation to each other, both in regard to their underlying magnitude and their form. Nevertheless, the ancients had a tendency to go even further: they invented another theory in regard to numbers [in relation to each other], no longer comparing randomly chosen numbers to each other, but judging each number in regard to (the sum of) its own parts.

34. The number, then, that is equal to (the sum of) its own parts they named a **perfect** number, because the sum of its parts neither falls short of nor exceeds it, such as the number 6. The number that is greater than the sum of its parts they called a **deficient** number, because the sum of its parts falls short of it, such as 8, but the number that is smaller than the sum of its parts, they referred to as an **abundant number**, because the sum of its parts exceeds it, such as 12.

35. Now that we have examined numbers both in relation to themselves and in relation to each other, let us say also a little concerning the theory of numbers in relation to both themselves and each other simultaneously.

Section 3:
36. Some (pairs of) numbers are both prime in themselves and (relatively) prime to each other, such as 3 and 5; others are both composite in themselves and in regard to each other, such as 6 and 9; yet others are composite in themselves and (relatively) prime to each other, such as 4 and 9.

37. Those numbers that are neither both prime in themselves nor both composite in themselves may be relatively prime to each other in one instance and in another the numbers may be found to be (relatively) composite. Sometimes a prime-and-incomposite number does not divide a composite number; that is, these two numbers are prime to each another, such as 3 and 8, but at other times the prime is a factor, such as with 3 and 6. For 3 is a common factor of both, for it not only divides 6 but also itself. [For since 3 divides 6 twice, it divides half of it once; that is, since 3 is equal to itself, it will also divide itself.] This, then, is the theory in regard to numbers with respect to both themselves and each other simultaneously.

Section 4:
38. The relations of equality and inequality that we distinguished in the preceding discussion have an immediate bearing on the topic of the so-called means and proportions because they are progressions composed of certain equal ratios. Previously we set these questions aside since they required special systematic treatment, but now let us say a little about them.

39. It is clear that their examination belongs exclusively to the theory of numbers in regard to each other and in regard to their underlying magnitude, as indeed also does the theory concerning the relation of equality and inequality; however, here we always have more terms, for a **ratio** occurs between two numbers, but a **pro-**

gression requires at least three terms.

40. If, then, there are three unequal numbers with the difference between the larger two equal to the difference between the smaller two, as is the case with 2, 4, and 6, they are said to be in **arithmetic progression** with regard to one another. They say that a progression is arithmetic whenever the middle term of three unequal numbers exceeds, and is exceeded, by the same amount.

41. But, if their differences are unequal, the difference of the larger two is the larger (in comparison with the difference of the smaller two) and this difference has the same ratio with the difference of the smaller two as any of the numbers has with one of the two remaining numbers, it is clear that this will either be the ratio that the largest has to the middle number or that the middle number has to the least. This ratio cannot be the ratio that the least has to the middle term, nor that which the middle has to the greatest, nor that which the least has to the greatest, since we supposed the difference of the greater terms to be the greater.

42. If the ratio of this difference to the difference of the smaller is the same as that which the greatest has to the middle term or the middle has to the smallest, the numbers are said to be in **geometric progression** with regard to one another, as is the case with 9, 6, and 4. For the difference of the larger numbers is one and a half times that of the smaller numbers, just as the greatest is to the middle and the middle to the least. For they say that a progression is geometric when, given three numbers that are unequal, the ratio that the first has to the second is also the ratio that the second has to the third number.

43. But if the ratio that the largest has to the smallest number is the one that the difference of the larger two has to the difference of the smaller two, the numbers are said to be in **harmonic progression** with regard to one another, as is the case with 6, 3, and 2, seeing that the greatest among these is thrice the least number and this is the same ratio as the difference of the greater pair to the lesser. For they say that a progression is harmonic when, given three unequal numbers, the ratio of the greatest to the least is as the difference of the greater two terms to the difference of the lesser.

44. It is necessary to note that the **means**, with respect to each type of progression, are able to fall between the same two extremes.

45. For example, the number 20 falls as the **geometric mean** between 10 and 40. For the ratio that 40 has to it, being double, is the same as the ratio it has to 10.

46. And 25 is the **arithmetic mean**, for 40 exceeds it by the same amount as it exceeds 10.

47. The number 16 is the **harmonic mean**, for the ratio that 40 has to 10, being four times as great, is the same ratio as the amount by which 40 exceeds 16 is to that by which 16 exceeds 10.

48. It is also necessary, then, to note that, whenever the means of the three types of progressions fall between the same two extremes, the geometric mean is not only the mean between the two extremes but also will be found to be the geometric mean of the other two means, as is the case with the numbers presently before us. The ratio that 25 has to 20, being one and a quarter times it, is the same ratio that 20 has to the number 16.

49. It is sufficient to speak here concerning these means and progressions only, because these [and such like them] are the ones thought to be worthy of attention by the ancients. But since the ancients introduced another theory in regard to numbers, comparing them with geometric shapes, we ought to speak a little about these things, beginning again afresh.

Section 5:
50. Some numbers, then, are said to be plane and some solid.

51. Those numbers that are formed from the product of any two numbers multiplied together are the **plane numbers**, such as the number 6 and the number 15, the former from twice 3, the latter formed from thrice 5. The numbers that are multiplied together are called the **sides** of the plane numbers.

52. Those numbers formed from three numbers multiplied together are called **solid numbers**, such as 24 and 125, the former being formed from twice 3 four times and the latter from five times 5 five times. The numbers being multiplied together are said to be their **sides**.

53. Some plane numbers are formed from two equal numbers, others from unequal.

54. Those, then, that are formed from equal numbers are called **squares**, such as 9 and 16, the first being thrice 3, the second, four times 4.

55. Those formed from unequal numbers are called **oblong**, such as 6 and 15.

56. These, then, are the (two) forms of plane numbers, but, of the solid numbers, there are four types. For either the three sides from which the numbers may have been formed are all equal to one another, two are equal and the remaining one greater, two are equal and the remaining side less or the three are unequal.

57. Those numbers, then, formed from equal sides are called **cubes**, such as 8 and 27, the former being twice 2 twice, the latter thrice 3 three times.

58. Those formed from two equal sides and from one greater side are called **columns**, such as 12 and 36: the former is formed from twice 2 three times, the latter thrice 3 four times.

59. Those formed from two equal sides and from one side that is smaller are named

tiles, such as 18 and 48: the former is formed from thrice 3 twice, the latter from four times 4 three times.

60. Those formed from three unequal sides are called **altars**, such as 24 and 105: the former is obtained from twice 3 four times, the latter thrice 5 seven times.

61. These are all the forms of plane and solid numbers formed from equal and unequal sides. It is not possible to find any more than these according to the principles of Euclid, the author of the *Elements*, or of Plato. For their theories regarding the geometric shapes among the numbers are only concerned with multiplication.

62. Plane and solid numbers are **similar** whenever their sides are in the same ratio to one another. This is the case for the plane numbers 6 and 54, where the former is formed from twice 3 and the latter from six times 9, and 2 is to 3 as 6 is to 9. Among the solid numbers, we have 24 and 192, the former formed from twice 3 four times and the latter from four times 6 eight times. For as 2 is to 3, thus 4 is to 6, and as 3 is to 4, thus 6 is to 8.

63. From what has been said, it is clear that one should speak of the theory of the geometrical forms of numbers both in themselves and in their relationships to each other, for a number is said to be plane or solid in relation to itself, but plane numbers or solid numbers are said to be similar only in comparison with each other.

64. In the theory in regard to numbers, we have spoken sufficiently as to what the forms of numbers are, in themselves and in their relation to each other separately, in themselves and in their relation to each other at the same time, and indeed also in their comparison with geometric figures.

65. But, as to the generations of numbers each with regard to their form, their peculiarities and the existence of an indefinite number of each form, all these things we will present in our *Elements of Arithmetic*. We will also prove in many different ways that one ought only to consider the geometric forms of numbers that have just now been discussed and consider whether the proportion between equal terms is to be regarded as arithmetic or harmonic, or if it should more naturally be regarded as geometric. We will discuss there many other indispensable questions, and indeed also most of the arithmetic that one finds in Plato. φ

Brief Notes on the Translation

Words enclosed in round brackets are not explicitly in the Greek but must be assumed to make sense of the passage. Only those numerals explicitly written in the Greek numeration system have been transcribed into modern numerals. Technical terms are printed in bold either at their first occurrence or when they are properly defined. In several places, a sentence is included in square brackets to indicate that, although comprehensible, it is possibly a gloss.

Section 1: The Classification of Numbers in Themselves (Boissonade 413.3-416.21)
1. Domninus takes the definition of the monad word for word from Euclid's

Elements, Book 7, Definition 1, (without the correction *ho* to *hên* that Tannery makes); accordingly, I have used Heath's literal rendering (retaining, however, "monad" for "unit"). (See Thomas L. Heath [tr.], *Euclid: The Thirteen Books of the Elements,* vol. II, 2nd ed. [New York: Dover, 1926].) The ancients did not regard the monad as a number, and the many varied definitions given by them seem to indicate the difficulty they had with the concept. Nicomachus avoids giving any definition at all. (For more details on the nature and definition of the monad, see Heath *Euclid* 279 and *History of Greek Mathematics* 70-71.)

2. The definition of "number" is also a cause of difficulty in antiquity (as in modern mathematics), and a variety of definitions were given. Here Domninus chooses not to follow the definition of Euclid but prefers that reported by Iamblichus to have been given by Thales. (See H. Pistelli [ed.], *In Nichomachi introductionem arithmeticam commentarius,* [Leipzig, 1894]; reprint, re-edited by V.Klein, with the title *Iamblichi, In Nicomachi Arithmeticam Introductionem Liber,* [Stuttgart: Teubner, 1975] 10, ll. 8-10.) Note that the list of definitions given in Nicomachus also includes that of Thales. The term "number," of course, always refers to a positive integer greater than one.

Domninus's definition of the sequence of numbers is similar to that of Iamblichus, who uses the term *propodismos,* "advancement," where Domninus uses *prokopê* (B. 413.3), here translated as "progression" but literally meaning "progress" or "advance." Nicomachus prefers the metaphor of "flowing." (See R.E. Taylor *Plato: the Man and his Works,* 6th ed., [Methuen, 1955] 506.) The term "to infinity" is the standard term used by Nicomachus (literally, "unto the boundless").

3. The Greek text here is corrupt. It appears that the words *dia to...phusei* (B. 413.7-8) have been interpolated from several lines below, where they are again repeated. A poor attempt, by someone not understanding the mathematics, may then have been made to "correct" the problem by inserting *temnein* and repeating *hoi men oun... duo isa* after *phusei.* I have translated what I believe to be the correct original.

Domninus's definition of even and odd numbers is precise yet simple, very close to that of Euclid (Bk. 7, Def. 6). Nicomachus offers a more lengthy definition, worth quoting for comparison: "The even is that which can be divided into equal parts without a unit intervening in the middle; the odd is that which cannot be divided into two equal parts because of the aforesaid intervention of a unit." (Bk. 1 7:2) He then goes on to give a more complicated definition, which he describes as Pythagorean. The key distinction between Nicomachus and Domninus here is the difference between the "intervention" of the monad and its "indivisibility."

Read *hautês* for *autês* (B. 413.11).

5. Note that the odd numbers start from 3 since the monad is not counted as a number.

6-8. The even numbers are further subdivided into evenly-even numbers, of the form 2^n, n a positive integer (this is not the same as Euclid's definition), evenly-odd numbers, and oddly-even numbers. These latter two terms are used without distinction by Domninus. Euclid, on the other hand, does not mention oddly-even numbers, while Nicomachus does distinguish in some detail between the evenly-odd numbers, of the form $2^{(2m+1)}$ and the oddly-even numbers, which have the form $2^{n(2m+1)}$ with n > 2. (See *Elements* Bk. 7, Def. 9 and Klein 27.) The subdivision also occurs in Plato (*Parmenides* 143E), where no clear distinction between the evenly-odd and the oddly-even is made.

9. The expression that I have translated, "which is the common beginning of all even numbers" is, literally, "as it were from the so-called common beginning of all even numbers." The dyad was not regarded by all ancient arithmeticians as a number, but rather as the principle of "the even."

On the gaps between evenly-odd numbers, compare Nicomachus (Bk. 1 9:5). Tannery has inserted *de* between *duadi* and *tou metaxu* (B. 414.19).

10-11. The odd numbers are divided into oddly-odd and prime-and-incomposite numbers. (These words appear linked also in Nicomachus, e.g. Bk.1 11:1, 2; Euclid uses the simple term *prôtos* [prime].) The linked words were sometimes counterbalanced by "secondary-and-incomposite." Although Theon curiously identifies primes as oddly-odd numbers, (see R. and D. Lawlor [tr.], *Theon of Smyrna: Mathematics Useful for the Understanding of Plato*, [San Diego, 1979] ch.6, p.15.), Euclid (Bk. 7, Def. 10) reserves that term for composite odd numbers. Nicomachus and Iamblichus do not use the term at all, but rather introduce three types of odd numbers: primes, odd numbers, and those composite in themselves but relatively prime to others (Nicomachus, Bk. 1 11). These last two classes clearly overlap: the classification is very awkward and is clearly rejected by Domninus.

Domninus's definition of prime numbers is slightly different from the others. Euclid and Theon say that primes have the monad as their only common factor, while Nicomachus says that the prime has one "part" (i.e. factor), which is the number itself. Domninus appears to exclude both as factors.

I accept Boissonade's *hopôsoun* for the manuscript's *hoposoun* (B. 414.27) but note Tannery's tempting suggestion *hoposousoun*.

12. Nicomachus (Bk. 1 11:2) and Iamblichus, unlike Euclid (Bk. 7, Def. 11) and Aristotle, do not appear to include 2 as a prime. (See Heath *Elements* Bk. 7, Def. 11.)

13. For *tou arithmou*, read *tois arithmois* (B. 415.8). The word translated "form" is *eidos*. Neoplatonic classification had aligned arithmetic with a study of "forms" or "kinds" of number and logistic with their "material" (*hylê*). (See Klein 7, 8, 11-13.)

14-18. On this section one may compare Archimedes's *Sand Reckoner*, in Thomas L. Heath [tr.], *The Works of Archimedes*, [New York: Dover, 1912] 227-9.) On "logistical theory," see Klein, n. 36 to Part 1, and Heath *History* 13ff., as well as the scholion to Plato *Charmides* 165 E. Neither Euclid, Nicomachus, nor Theon discuss the logistical aspect of number. The four classes called "tetrads" go back to a now-lost arithmetical work by Apollonius. The essence of it, however, is presented in Pappus, Book II. For further details, see Heath *History* 40.

In chapter 14, read *monasi* for *monadi* (B. 415.12) and insert *mechris enneados* after *apo monados* and *en chiliasi de, hoi tôn en hekatontasi dekaplasioi* (B. 415.15), which has obviously fallen out. In chapter 15 (B. 415.22), read 'β for μ'. In chapter 16, we need the words "thousands of myriads" for completeness, but the Greek words seem to have dropped out of the text. Tannery's "Le Manuel" translates as though they were present, but he makes no mention of this in his "Notes Critiques." In chapter 17, read *logistikês* (B. 416.6) for the obviously incorrect *logikês* and delete the period after *epistêsomen* (B. 416. 8). There appears to be a lacuna in chapter 18 (after "evenly-odd or oddly-even"), unnoticed by Tannery, where one would have expected a recapitulation of the evenly-even numbers.

19. This is a transition passage between the two sections. The remark about "starting afresh" occurs again in chapter 49 and is a commonplace in Nicomachus (e.g. Bk.1 3:1, 14:1). I prefer the reading *legômen* to the manuscript's *legomen* (B. 416.22).

Section 2: The Classification of Numbers in Regard to Each Other. (Boissonade 416.21–422.5)

20-21. One should compare Euclid, Book 7, Definitions 12-14. Observe that Nicomachus does not make as sharp a distinction between numbers in relation to themselves and in relation to each other as Domninus does (see Klein 26-27), and Theon does not mention the distinction at all. Thus, the classification of odd numbers in Nicomachus was rejected by Domninus since one of the types depended on a relational comparison not intrinsic to the number itself. The words translated as "have a common factor" are, literally, "are measured in common measure by the same number."

23. The strange concern with the uniqueness of equality and its being incapable of subdivision appear also in Nicomachus (Bk.1 17:3-5).

24-32. The "remark" in chapter 24 is similar to that in Euclid, Book 7, Definitions 3, 4, on which see Heath *Euclid* 280. Gow 75 n.1 remarks that the term *merê* (parts) is very inconvenient. I have translated it as "proper fraction" (except at the end of chapter 24), a term used by Heath to mean a fraction whose numerator is greater than one. Thus, 6 is a *meros* (part) = "factor" of 12, whereas 9 is *merê* (parts) = "proper fraction" of 12, being 3/4 of it.

To summarize the terms, if a < b, then the relationship between and a and b is called:

multiple if a is a factor of b;
superparticular if b = 1 + 1/a;
superpartient if b = 1 + n/a, n > 1;
multiple-superparticular if b = m + 1/a, for m > 1;
multiple-superpartient if b = m + n/a, for n, m > 1.

The names of the relationships between b and a are simply obtained by adding the prefix "sub." On these ten relationships, see Heath *History* 101ff. and Nicomachus Bk.1 17:7ff.

I have retained the Latin terms such as "sesquialter," etc., since they are used in the standard translations of Nicomachus and Theon, and in Heath. It is convenient to give a list of those appearing here and their arithmetic equivalents:

sesquialter = 1-1/2;
sesquitertian = 1-1/3;
superbitertian = 1-2/3;
supertriquintian = 1-3/5;
duplex-sesquialter = 2-1/2;
quadruplex-sesquitertian = 4-1/3;
duplex-superbitertian = 2-2/3;
triplex-superbiquintian = 3-2/5 .

In chapter 24 (B. 417.21), read *Dei de tou p.a. logou*, delete the period after *hêmas* and read (B. 417.22) *hoti dê* (for *de*) *pas arithmou*. In chapter 25 (B. 418.2), it is not necessary to alter *metrei* to *metrôn*; however, read *duo trita estin* for *duo tria* *estin*. In chapter 27 (B. 419.5) I prefer to retain Boissonade's emendation *pantôs* for *pantos* and also read *monadôn* for *monados* at line 9. At the end of chapter 29 (B. 420.2), read *pempta* for *ε'*. For details of these changes, see Tannery "Notes Critiques." In chapter 31 (B. 420.20), read *merê* instead of *meros*. The *de* has been correctly altered to *dê* by Boissonade.

33-35. Domninus rounds off this section but also introduces the notions of perfect, abundant, and deficient numbers. Euclid (Bk.7, Def. 22) mentions perfect numbers (and gives the method of construction for even perfect numbers) but does not give the other two categories. These appear in both Nicomachus (Bk. 1 16) and Theon (Lawlor 30-31). The latter part of chapter 33 is somewhat confusing, since it refers to this classification under the heading of "numbers in relation to each other," which I have placed in square brackets since it contradicts what follows. Tannery has correctly interchanged *elattona* and *meizona* in chapter 34 (B. 421.20 and 422.1), so that the definitions are consistent with those of Euclid and Nicomachus. The reading *ιε'* (B. 421.20) as an example of a perfect number is clearly wrong and should be replaced with *σ'*. I do not find Tannery's reconstruction leading to the inclusion of both 6 and 28 convincing.

Section 3: Numbers in Regard to Themselves and Each Other Simultaneously. (Boissonade 422.5–423.2).

36-37. On this section, compare Nicomachus, Book 1 11-13, the much briefer Euclid, Book 7, Definition 14, and Heath *Euclid* loc. cit. The sentence in square brackets was regarded as suspect by Tannery since it implies that 3 is a factor of itself, which is not in keeping with Domninus's general viewpoint. Read *kath' heautous* for *pros allêlous* in chapter 37 (B. 422.12). For *auton* read *autou* in the second last line and *heautou* for *autou* in the last line of Boissonade 422.

Section 4: The Section on Means and Proportions. (Boissonade 423.6–426.3) (My division of the text differs slightly here from that of Hultsch [see A. Pauly and G. Wissowa *Real-Encyclopädie V*, (Stuttgart: 1903) s.v. "Domninos."])

38-39. Ratio (relating to two terms) and progression (relating to three or more terms) are seen as part of the classification of equality and inequality. No mention is made of the distinction between "continuous proportion" and "discrete proportion." (See Nicomachus Bk.2 21 and Euclid Bk.5, Def. 8-10.) In chapter 38 (B. 423.4-6), the text is corrupt in several places. One should read *sunecheis ontas tôi peri (tôn) legomenôn mesotêtôn te kai analogiôn topôi, eita* (Tannery prefers *eite* here, but seems to translate as though it were *eita*), *analogôs ek tinôn tôn autôn sunkeitai logôn* (comma or even full stop) *parethemetha*. In chapter 39 (B. 423.10), read *monôs* for *moriôn*.

40. Arithmetic progressions were regarded as the most basic of progressions, since any sequence of consecutive numbers 2, 3, 4, 5,..., n is always such a progression. Add *kai δ'* after *β'* (B. 423.19), which has clearly fallen out.

41-42. The long-winded discussion in chapter 41 is simply stating that we are considering progressions of the form a, b, c (with $a < b < c$) such that $c-b:b-a = b:a = c:b$ and that this cannot be the same as a:b, c:b, or a:c.

In chapter 42, I have had to alter the word order severely to make the English sentence comprehensible, since the example 9, 6, 4 is given in detail between the definition of a geometric progression and its name. I have had to do a similar rearrangement in chapter 43 on harmonic progressions. In chapter 41, insert *eien* between *ei* *de* and *anisoi* (B. 423.22), alter *echei* to *echoi* and delete the following comma (B. 423.24). The word *megistos* (B. 424.1) should be *mesos,* while the last clause of the chapter should read *meizonos* (for *meizô te*) *einai hupokeimenês tês tôn meizonôn* (for *elattonôn*) *diaphoras* (B. 424.4-5).

In chapter 42, read *mesos* for *megistos* (B. 424.6), replace the full stops before *hêmiolos* and *hôsper* with commas (B. 424.9, 11), read *hôsper dê* (instead of *de*), and also delete the words *hoion arithmêtikoi* (B. 424.12), which make no sense here.

43. The harmonic progression is c-b:b-a = c:a. It is more familiar today as the reciprocal of the arithmetic mean of the reciprocals, i.e. $1/b = (1/a + 1/c)/2$. In the fifth line from the bottom of p.424, Tannery says to read *ho te* for *hote*; however, *hote* can have the meaning "seeing that" (see LSJ loc. cit.), which makes perfect sense, and I prefer to retain the manuscript reading. Read *hêi* for *ê* in the last sentence (B. 425.1).

44-48. The fact that the geometric mean of two numbers is also the geometric mean of the arithmetic and harmonic means of the given numbers does not appear in any of the earlier (surviving) arithmetical writers but was probably known before Domninus's time.

In chapter 46 κα' should, of course, be *kai* (B. 425.11), and one should delete the period after *heurethêsetai* (B. 425.20). In chapter 47 (B. 425.15), read ισ' for ιε'. In chapter 48 (B. 425.17), read *autôn* for *autou.*

49. There seems some confusion here, although Tannery appears not to have noticed it. Domninus says that it suffices to speak only (*monôn*) of the three means that were worthy of study by the ancients. Nicomachus adds seven more means, whose value Domninus appears to be politely rejecting. At the end of this sentence, however, we have the words *hautai te eisi kai toiautai* (these and such like them), presumably refering to these other means, which I believe to be a later addition. I have therefore placed these words in square brackets.

Section 5: The Geometric Classification of Numbers. (Boissonade 426.4–428.8)

50-55. On the classification into plane and solid numbers, see Euclid, Book 7, Definitions 16 and 17. Domninus subdivides plane numbers into square numbers and oblong numbers, following Euclid. As usual, Nicomachus has a more elaborate division, adding numbers of the form n(n+1) as a special category (Bk.2 6).

In chapter 52 (B. 426.13), read *ek tou pentakis ε' pentakis* instead of *ek tou κε' pentakis.*

56-60. There was some variation in the names given to the different types of solid numbers. Those called *bômiskoi* (altars) are also known as*sphêkiskoi* (stakes) and *sphêniskoi* (wedges); *stêlides* (columns) are also known as *dokides* (beams). (Tannery "Le Manuel" 372, says of *stêlides,* "n'a, que je sache, été employée par aucun autre auteur;" however, the term does appear in Iamblichus [Klein 95, l. 9]. See Nicomachus Bk.2 15,16, and Heath *History* 107ff.) Domninus appears to be following the Iamblichus source here, since he uses the same terminology as Iamblichus does. The first words of Boissonade's p.427 should read *hoi men* for *ei men.* In chapter 58 (B. 427.5), read *meizonos de tês mias pleuras* for *meizones de, tois pollois.* In chapter 60, read ρε' for οε' and *heptakis* for *pentakis* (B. 427.13, 17).

61-64. In chapter 61, add *stereôn* before *eidê* and *ho* before *stoicheiôtês* (B.427.16, 17). In chapter 62, for *allêlous* read *allêlas* (B. 427.22), and, in chapter 63, we need *theôrian* instead of *theôrias* (B. 428.4). Nicomachus does not treat similar solid

numbers. In chapter 64 (B. 428.12), read *geômetrika* for *megista*.
65. The *Elements of Arithmetic* has not survived, if indeed it was ever written in the first place. One should add *dei* or *chrei* after *schêmata* or thereabouts (B. 428.18).

Notes

[1] Paul Tannery, "Le Manuel d' Introduction Arithmétique du philosophe Domninos de Larissa,"*Revue des Études Grecques* (1906), 360-382.

[2] The Greek text of this passage may be found in Adolf Westerman (ed.), *Biographi Graeci Minores*, rev. ed., (Amsterdam: 1964) 417.

[3] J. Fr. Boissonade (ed.), *Marini Vita Procli* (Lipsiae: 1814) 20.

[4] Dominic O'Meara, *Pythagoras Revived, Mathematics and Philosophy in Late Antiquity* (Oxford: Oxford University Press, 1989), 144.

[5] James Gow, *A Short History of Greek Mathematics* (Cambridge: University Press, 1884) 45.

[6] Tannery "Le Manuel."

[7] Gow 142; Thomas L. Heath, *A History of Greek Mathematics*, (New York: Dover, 1981) Vol.1: 538.

[8] J.Fr. Boissonade, ed., *Anecdota Graeca* , vol. IV (Paris: 1833; reprint, Hildesheim: Olms, 1962).

[9] Tannery "Notes Critiques sur Domninos," *Revue de philologie* (1885) 129-37.

[10] Jacob Klein, trans. *Greek Mathematical Thought and the Origin of Algebra*, (Cambridge: M.I.T. Press, 1968; reprint, New York: Dover, 1992); originally published as "Die griechische Logistik und die Entstehung der Algebra"*Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, Abteilung B: *Studien*, 3 (1934): fasc. 1, 18-105; fasc. 2, 122-235.

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